HIGHWAY SAFETY MANUAL:
SPATIAL HETEROGENEITY, TRANSFERABILITY, & NON-LINEARITIES IN SAFETY PERFORMANCE FUNCTIONS

Final Report, Year 2

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ASAD KHATTAK, BEHRAM WALI & JUN LIU

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16. Abstract: The Highway Safety Manual (HSM) is extensively used by transportation agencies to identify sites with safety problems and select countermeasures aimed at reducing the likelihood of crashes. Safety Performance Functions (SPFs), as the core tools developed in HSM, are used to estimate expected crash frequency of a network, facility or individual site, given the level of traffic exposure and site attributes. Moreover, SPFs that can accurately estimate crash frequencies are critical to local and state transportation agencies due to their ability to identify sites with potential safety concerns. While HSM SPFs are useful and are extensively applied by practitioners for safety countermeasure design, accurate and reliable prediction of crashes is constrained by several methodological issues, some of which can fortunately be addressed by fusing recent advances in methodological techniques, data science, and computational power. This report presents research activities undertaken to explicitly address two important issues pertaining to SPFs, 1) investigating spatial heterogeneity and transferability of single statewide SPFs, and 2) analyzing nonlinear dependencies between crash frequencies and key factors. By developing and applying advanced techniques to address methodological issues, more effective context-driven safety countermeasures can be evaluated to reduce transportation injuries and deaths.		
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EXECUTIVE SUMMARY

The Highway Safety Manual (HSM) is used extensively by transportation agencies to identify factors contributing to safety outcomes and select countermeasures aimed at reducing the likelihood of crashes. Safety Performance Functions (SPFs), as the core tools developed in HSM, are used to estimate expected crash frequency of a network, facility or individual site, given the amount of traffic exposure and site attributes. Moreover, SPFs that can accurately estimate crashes are critical to local and state transportation agencies due to its ability to identify regions with potential safety concerns. While HSM SPFs are useful and are extensively applied by practitioners for safety countermeasure design, accurate and reliable prediction of crashes is constrained by several methodological issues, which can fortunately be addressed by fusing recent advances in methodological techniques and computational power. This report presents the research activities undertaken to explicitly address two important issues pertaining to SPFs, 1) investigating spatial heterogeneity and transferability of single statewide SPF, and 2) analyzing nonlinear dependencies between crash frequencies and key factors. By development and application of advanced econometric techniques to address important fundamental methodological issues, it is expected that more effective safety countermeasures can be evaluated to substantially reduce road traffic injuries and fatalities.

First, Safety Performance Functions (SPFs) provide a basis for identifying locations where countermeasures can be effective. While SPFs in the Highway Safety Manual (HSM) were calibrated based on data from select states, calibration factors can be developed to localize SPFs to other states. Calibration factors typically provide a coarse adjustment—time and space stationarity of associations between crash frequencies and various factors is still assumed, implying that the SPF functional form is transferable. However, with increasing availability of statewide geo-referenced safety data, new spatial analysis methods, and increasing computational power, it is possible to relax the stationarity assumption. Specifically, to address spatial heterogeneity in SPFs, this study proposes relaxing SPFs (referring to them as Localized SPFs) that can be developed by using sophisticated geo-spatial modeling techniques that allow correlates of crash frequencies to vary in space. For demonstration, a 2013 geo-referenced freeway crash and traffic database from Virginia is used. As a potential methodological alternative, crash frequencies are predicted by estimating Geographically Weighted Negative Binomial Regressions. This model significantly outperforms the traditional negative binomial model in terms of model goodness-of-fit, providing a better and fuller understanding of spatial variations in modeled relationships. Our study results uncover significant spatial variations in parameter estimates for Annual Average Daily Traffic (AADT) and segment length. Ignoring such variations can result in prediction errors. The results indicate low transferability of a single statewide SPF highlighting the importance of developing L-SPFs. From a practical standpoint, L-SPFs can better predict crash frequencies and support prioritizing safety improvements in specific locations.

Second, for practical considerations and in many cases the difficulty to collect detailed crash-related data, AADT and segment length are often used as the main correlates for predicting crash frequencies on segments. Typically, crash frequencies are assumed to linearly depend on traffic exposure related factors which may not realistically represent the underlying complexity embedded in crash data generated by physical and social elements of transportation systems. Thus, the objective is to investigate and quantify nonlinear dependencies of crash frequency on traffic exposure related factors. Using crash data collected on rural two-lane two-way roads in Tennessee, total crashes and total injury crashes were modeled using Negative Binomial Generalized Additive Models (NBGAMs) that are well-suited for conceptualizing non-linear relationships. In addition, including too few explanatory factors (such as AADT and segment length only) in crash frequency modeling may lead to omitted variables bias, and in such cases the nonlinearity may be an outgrowth of missing information on important variables. To address this issue, additional data on important correlates are collected and incorporated in NBGAM framework. The modeling results show that the relationship between crash frequencies (total crashes and total injury crashes) and AADT is clearly non-linear. Importantly, the non-linear dependency of crash frequencies on segment length is more complex than its dependence on AADT. The goodness of fit measures indicates the promising potential of NBGAMs in approximating non-linear dependencies of crash frequencies on associated factors. Important practical implications of results are presented with respect to rural two-lane two-way road safety.

Overall, by addressing important fundamental methodological issues pertaining to HSM SPFs, the research activities have focused on application of new frameworks that contribute to methodological enhancements of Highway Safety Manual procedures. During the reporting period, the above mentioned activities led to preparation of two full-length research papers, which will be submitted for publication and presentation review.

1. Liu J., A. Khattak, B. Wali, Do Safety Performance Functions Vary Across Space? Application of Geographically Weighted Regressions. To be submitted to a transportation conference and a safety journal for publication review.
2. Khattak A., B. Wali, X. Li, Exploring Non-Linear Dependencies in Correlates of Roadway Crashes. To be submitted to a transportation conference and a safety journal for publication review.

1. DO SAFETY PERFORMANCE FUNCTIONS VARY ACROSS SPACE?

Application of Geographically Weighted Regressions[1]

1.1. INTRODUCTION

The Highway Safety Manual (HSM) is extensively used by transportation agencies to identify factors contributing to safety outcomes and select countermeasures aimed at reducing the likelihood of crashes (*1*). Safety Performance Functions (SPFs), as the core tools developed in HSM, are used to estimate expected crash frequency of a network, facility or individual site, given the amount of traffic exposure, i.e., traffic volume. Moreover, SPFs that can accurately estimate crashes are critical to local and state transportation agencies due to its ability to identify regions with potential safety concerns (*2*). Since crash occurrence/frequency and the associated under- and over-dispersion in crash data can vary significantly across jurisdictions, it is important to calibrate HSM SPFs for specific jurisdictions (*3*). The need for calibrating HSM SPFs to specific jurisdictions is clearly recognized by the American Association of State Highway and Transportation Officials (AASHTO) due to variations in factors associated with safety. Such factors include road geometry and conditions, environmental factors, geographic characteristics, crash characteristics, reporting thresholds, all of which can be unique to specific jurisdictions (*1*).

From a methodological perspective, given the count nature of crash frequencies, Poisson and Negative Binomial regression models are used as state-of-the-art for developing jurisdiction-specific SPFs (*1, 4, 5*). However, the aforementioned modeling techniques generally assume time and space stationarity of crash frequencies and the factors that may be associated with crash frequencies. For instance, a single coefficient is estimated for the relationship between Annual Average Daily Traffic (AADT) and crash frequency for the entire jurisdiction. Despite the fact that jurisdiction-specific SPFs (as compared to HSM SPFs) can better represent local conditions at hand, traffic crash

[1] Material in this section is based on: Liu J., A. Khattak, B. Wali, Do Safety Performance Functions Vary Across Space? Application of Geographically Weighted Regressions. To be submitted to a transportation conference and a safety journal for publication review.

frequencies and associated factors (such as traffic volumes) can vary significantly across similar, or even identical, road geometry and conditions within the jurisdiction where a single SPF is estimated (*6*). For instance, crash data and associated factors (such as traffic volumes) are location-referenced, and this along with the spatiotemporal nature of traffic crashes and spatial dependence between crash observations, can result in spatial heterogeneity in the relationships that are modeled (*7-11*). In addition to this, for ease of practical applicability, both HSM SPFs and a majority of jurisdiction-specific SPFs developed by many local agencies have AADT and segment length as two critical explanatory variables. Many factors that are likely to contribute to crash frequency are not observable, or unknown from the data at hand. Methodologically, as explained in Mannering and Bhat (*12*) and Mannering et al (*13*), the presence of unobserved factors and its correlation with observed factors (such as AADT and segment length) can ultimately result in important issues related to unobserved heterogeneity and/or Parsimonious vs. Fully Specified Models, potentially resulting in highly inconsistent and biased parameter estimates.

Using geo-referenced safety data, the present study aims to address spatial heterogeneity (or variation) in freeway crash data by proposing relaxed SPFs (referred to as Localized SPFs), which allow the associations between crash frequencies and correlates to vary in space. Note that unobserved heterogeneity (resulting from several unobserved factors) in crash frequency modeling has been successfully captured through random-parameter models (*14-16*). However, we posit that crash data are increasingly location-referenced, and therefore we should explicitly utilize geo-referenced component of crash data to capture spatial heterogeneity. Thus, from a methodological standpoint, as we will demonstrate, the possibility of accounting for spatial heterogeneity by allowing some or all parameters to vary spatially.

1.2. LITERATURE REVIEW

1.2.1. Methodological alternatives

For decades, researchers have used a wide variety of rigorous statistical tools in order to seek better understanding of factors associated with crash frequencies. While a wide variety of statistical methods have appeared, a thorough review of literature reveals Poisson and Negative Binomial/Poisson-Gamma as the two potential state-of-the art alternatives (*1, 17*). For a complete review of various methodological challenges, issues and modeling alternatives, see Lord and Mannering (*17*).

A few recent studies have explicitly focused on SPF transferability issues by examining the viability of national level SPFs for local jurisdictions (*18-20*). Specifically, by using various statistical measures such as transfer index, Poisson-Gamma and Bayesian averaging approaches, the studies have concluded superior transferability potential of estimated models by using data from different states (*19*) and different countries (*18, 20*). In addition to calibration factors (discussed below), a recent study by Farid et al proposed Modified Empirical Bayes to improve transferability of SPFs to local jurisdictions (*19*).

1.2.2. Calibration of SPFs

Several researchers have successfully attempted to implement and calibrate HSM crash prediction models/SPFs by using jurisdiction-specific data in various states including (out of many) Louisiana, Texas, Oregon, Illinois, Missouri, Utah, South Dakota, and North Carolina (*4, 21-27*). The results from these studies collectively document, as recommended by HSM too, that customized state-specific models/SPFs outperform the HSM SPFs in predicting state-specific crash frequencies.

1.2.3. Jurisdiction-specific Freeway SPFs and Calibration factors

A detailed methodology for prediction of crash frequencies on freeway segments is (currently) not officially included in HSM. However, Appendix C of HSM presents a proposed chapter for freeway crash prediction (*1, 23*). A relatively broad study by Sun et al. (*23*) investigated HSM default SPFs for a wide variety of roadway types including rural two-lane undivided segments, rural multilane divided segments, urban arterials, freeway segments, urban signalized intersections, and urban unsignalized intersections (*23*). The results indicate that, for rural four-lane freeway segments, the number of property damage only (PDO) crashes (both single- and multiple-vehicle) observed in Missouri were greater than the predicted crashes by HSM SPFs (*23*). Contrarily, fatality and injury crashes were over-predicted by HSM SPFs (*23*). The calibration of HSM SPFs using data from North Carolina resulted in similar outcomes (*4*). Jurisdiction-specific SPFs for 16 roadway types including rural and urban freeways were estimated due to significant over- and/or under-prediction of actual crashes by HSM SPFs (*4*). The results obtained from afore-mentioned studies may be an outgrowth of the fact that crash characteristics, environmental factors, and geographic factors of different jurisdictions vary substantially than those for which the HSM SPFs are originally developed.

Kweon and Lim (*2*) developed SPFs for 14 freeway and multilane highway segments in Virginia. The study concluded that default HSM SPFs do not properly represent the actual relationship between crash frequencies and AADT on Virginia multilane highways and freeways (*2*). To explicitly address the issue of spatial non-stationarity in the associations between several variables and crash frequency, recent studies have used Geo-spatial modeling techniques (*6, 9*). For instance, Xu and Huang (*9*) addressed spatially heterogeneous associations between key correlates and crash frequency by using random-parameter negative binomial and Geographically Weighted Poisson regressions (GWPR). By using statewide data from Florida, the study modeled total crash frequency as a function of intersection/road length density, population density, median household income, and percent of road segments with different speed limits, and concluded superior statistical performance of GWPR in capturing spatial non-stationarity as compared to random-parameter negative binomial model (*9*). Likewise, the study by Rhee et al (*6*) investigated spatially correlated traffic crashes and associated correlates through advanced spatial modeling techniques. The results showed superior statistical performance of GWR in capturing localization of correlates.

Studies provide valuable insights for understanding spatially varying associations between disaggregate level variables and crash frequency. However, strictly from the HSM crash prediction framework perspective, the studies did not focus on modeling spatially non-stationary relationships between AADT, freeway-segment length, and crash frequency.

Moreover, traffic exposure (such as AADT and segment length) are considered key predictors of crash frequencies and are widely used by transportation professionals for predicting crash occurrence at a particular site. Thus, in context of SPFs, an understanding of spatially varying relationships between key exposure factors and crash frequencies has significant potential to develop localized-SPFs that can potentially make more accurate crash predictions at individual sites.

1.3. METHODOLOGY

To explore transferability (spatial stability) of global SPFs to specific jurisdictions, this study proposes application of Localized SPFs (L-SPFs) that are developed using geospatial modeling techniques. As a promising methodological approach, Geographically Weighted Negative Binomial Regression (GWNBR) model is estimated and it is compared with the conventional (fixed parameter) negative binomial model. GWNBR models can test whether a relationship is stable or it varies substantially over space. By exploiting the growing amount of geo-referenced data and modern computational power, the L-SPFs developed through rigorous geo-spatial modeling techniques are able to account for spatial heterogeneity, which is reflected in the variation of relationships between crash frequency and associated factors.

The focus of this study is to suggest methodological advances to HSM procedures, which is timely and original given the high levels of safety costs and the need to implement effective countermeasures. The L-SPFs are important in prioritizing the safety improvements for specific roadway locations if the spatially varying patterns highlight serious and significant risks at these locations. The advantage of L-SPFs is that more realistic predictions of crashes can be obtained that can better identify hazardous sites, and appropriate countermeasures can be developed. To the best of our knowledge, base crash frequencies as a function of AADT and segment length have not been estimated using rigorous spatial tools such as GWNBR. Note that GWNBR provides the mechanism for making base crash frequency predictions that are customized to specific locations. The proposed spatial framework has the potential to change the current state of practice, i.e.

calibration of global models used in HSM to match local conditions. Given that expected number of crashes (at base conditions) at various sites can be determined more accurately with the proposed methodology, the need for current HSM calibration procedure can diminish or be eliminated altogether.

1.3.1. Data

This study used a statewide crash database to demonstrate the development of Localized SPFs. The data was obtained from the Virginia Department of Transportation (VDOT). It includes all types of traffic crashes that occurred within the Commonwealth of Virginia in 2013. The crash database contains individual crashes reported using the Virginia Police Crash Report. As mentioned earlier, the scope of this study is limited to freeway segments, and the SPFs are for crash frequency, therefore all freeway crashes in the databases were counted for homogeneous freeway segments, which are defined and divided in VDOT's traffic count data program (http://www.virginiadot.org/info/ct-trafficcounts.asp). The freeway crashes are linked with the freeway segments through two pieces of information that are available in both datasets: 1) the mile post and 2) geo-coordinates (longitude and latitude), as some crashes were reported with either of them, or both. Note that, crash observations with missing geo-referenced information were removed from the traffic counts for homogeneous freeway segments. In addition, the underreported crashes due to the small or minimal damage may also cause the crash count missing or under-counted for some segments. After data reduction (limited to freeway segments) and data cleaning (removing observations with missing information), this study analyzed traffic crash frequencies for 2,116 homogeneous freeway segments, and 15,426 crashes were counted for these segments. Note that the coefficient of variation (CV) (standard deviation/mean) of observed crash data is 2.12 that highlights significant over-dispersion in the data, and utilizing a one-size-fits-all sample-size of 30-50 segments (as recommended by HSM) may not be appropriate (*3*). According to the study by Shirazi et al. (*3*), in order to fulfill 90% confidence level for crash data with CV around 2.2, the study recommended a minimum sample size of 1,000 segments, and thus a relatively larger sample size (N = 2,116) is used in the current study. Table 1 presents the descriptive statistics of variables used in standard SPFs (crash frequency, traffic volume and segment length). In HSM, the log-transformation of traffic volume and segments is used in safety performance functions, as provided in Table 1. Figure 1 shows: (a) traffic volume of freeways in Virginia (denoted by the width of lines); (b) the spatial distribution of individual crashes (one point indicating one single crash) and spatial density; and (c) the spatial distribution of crash frequency on freeways across Virginia.

Table 1 Descriptive Statistics of Key Variables in 2013 Virginia Crashes on Freeways

Variable	Valid N	Mean	Std. Dev.	Min	Max
Crash Frequency	2,116	7.290	15.504	0	184
Traffic Volume (AADT)	2,116	42610.70	51272.94	30	255000
Segment Length (mile)	2,116	1.169	1.557	0.020	9.390
Ln (AADT)	2,116	9.611	1.767	3.401	12.449
Ln (Segment Length)	2,116	-0.643	1.286	-3.912	2.240

Notes:

1. AADT = Annual Average Daily Traffic
2. Std. Dev. = Standard Deviation

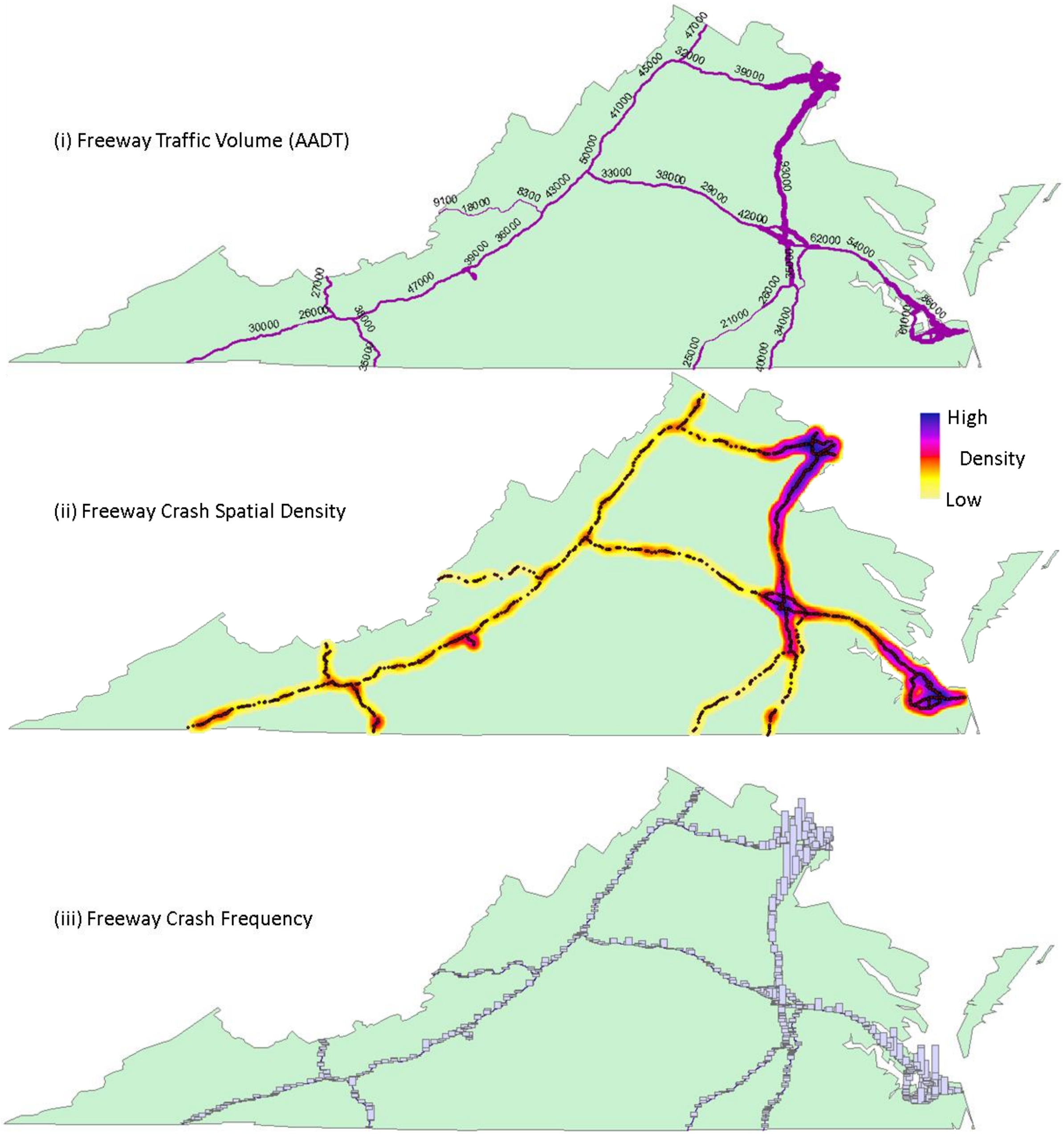

Figure 1 Spatial distribution of freeway traffic volumes, freeway spatial crash density, and freeway crash frequency

1.3.2. Standard SPF – Negative Binomial Model

As recommended by HSM, the standard or most common form of a SPF for roadway segments is described below (*1, 2, 4*):

$$N_{SPF} = L \times \mathrm{e}^{a+b\times\ln(AADT)} \text{ or } N_{SPF} = \mathrm{e}^{a+b\times\ln(AADT)+\ln(L)} \quad (1)$$

Where N_{SPF} is the predicted number of crashes on a segment; L is the length of the segment; $AADT$ is annual average daily traffic volume; a and *b* are regression coefficients to be estimated using historical crash data. In Equation (1), the segment length *L* is included as a multiplier, which assumes that the crash frequency on a segment is simply proportional to the segment length. However, this assumption may be inappropriate in some cases. Traveling on a road segment, a driver experiences homogeneous road conditions (including the number of lanes, the shoulders, etc.)

Driving on a relatively longer road segment with unchanging circumstances may be different from driving on a relatively shorter road segment with frequent variation of circumstances. Therefore, another common form of SPFs is also suggested by transportation professionals:

$$N_{SPF} = \mathrm{e}^{a+b\times\ln(AADT)+c\times\ln(L)} \quad (2)$$

Where c is a parameter indicating the relationship between crash frequency and segment length. If the estimate of *c* is close to 1, then the Equation (2) is identical to equation (1). If *c* is significantly different from 1, then it shows that the road segment length is not simply proportional in relation to crash frequencies.

Multiple regression models are estimated, providing parameters (*a*, *b* and *c*) of the SPFs. It is assumed, in HSM, that crash frequencies follow a negative binomial (NB) distribution (*1, 4*). The negative binomial distribution is an extension (capturing over-dispersion) of the Poisson model (*5*):

$$Y_i \sim NB\ (N_{SPF\ i}, \alpha) \quad (3)$$

Where Y_i is the observed crash frequency on a segment; $N_{SPF\ i}$ is the expected crash frequency; and α is the NB over-dispersion parameter. A larger value of α implies greater over-dispersion in data. If α = 0, then the data follows a Poisson distribution (where mean = variance). In such a situation, the Poisson and Negative Binomial model provide identical estimates of parameters (*a*, *b* and *c*). If α is significantly greater than 0, a NB model is preferred. Formally, $N_{SPF\ i}$ can be viewed as a log link function of a set of independent variables:

$$Ln\ (N_{SPF\ i}) = a + b \times \ln(AADT_i) + c \times \ln(L_i) \tag{4}$$

To reflect the localization of highway safety performance, separate SPFs can be developed for each jurisdiction through regressing the local crash data on local traffic volumes and segment lengths (or by using HSM calibration procedures). However, the parameters *a*, *b* and *c* may vary across jurisdiction-specific SPFs, but they are assumed to be stationary within a jurisdiction.

1.3.3. Localized SPF – Geographically Weighted Negative Binomial Regression

To overcome the borders of jurisdictions while developing the SPFs to reflect the local roadway conditions, this study used a geo-spatial modeling technique to examine spatial heterogeneity in highway safety performance simultaneously. The technique produces SPFs that are Localized (i.e., L-SPFs) in continuous space rather than discrete space of jurisdictions. Geographically Weighted Negative Binomial Regression (GWNBR) was employed in this study. GWNBR assumes spatial non-stationarity and allows different associations (i.e., parameters *a*, *b* and *c*) to exist at different observations in continuous space. In the process of GWNBR, there is a local NB model estimated for each observation's site based on a subgroup of observations, which are geographically centered at the location of a target observation. Observations for the local NB models are selected based on their geographical distance to the target observation (i.e., the central observation). GWNBR has a form that is an extension of the traditional NB model, Equation (4). The form can be viewed as L-SPFs written as (*28*):

$$Log\ (N_{SPF\ i}) = a(u_i, v_i) + b(u_i, v_i) \times \ln(AADT_i) + c(u_i, v_i) \times \ln(L_i) \tag{5}$$

Where, $a(u_i, v_i)$, $b(u_i, v_i)$ and $c(u_i, v_i)$ are the estimates of the local parameters in L-SPFs for *i*th road segment whose location is denoted by (u_i, v_i). Subgroup observations in local NB models are assigned according to their spatial distance to the central observation (*28*). Adaptive bi-square function is often used to determine weights for each observations (*28*):

$$w_j(u_i, v_i) = \left(1 - \left(\frac{d_{ij}}{d_{bandwidth}}\right)^2\right)^2 \tag{6}$$

Where, $w_j(u_i, v_i)$ is the weight for *j*th observation within the subgroup, which is centered at *i*th road segment in the overall sample (e.g., nationwide, statewide or any jurisdictional region); d_{ij} is the geographical distance of *j*th observation from the center of the subgroup (i.e., the location of *i*th road segment); $d_{bandwidth}$ is bandwidth of the subgroup, equal to the geographical distance of the farthest observation within the subgroup from the center. There are *m* observations in each subgroup, *j* = 1, 2, …, *m*, and *m* <= *n* where *n* is the total number of observations. Though GWNBR is better at untangling the spatial variations across the space, the modeling results may show that the parameters $a(u_i, v_i)$, $b(u_i, v_i)$ and $c(u_i, v_i)$ do not vary substantially across the study area. In this case, the L-SPFs developed based on GWNBR models are close to the traditional or global SPFs.

1.3.4. Marginal Effects

The marginal effects, *MFX*, for traffic volume and segment length were computed to provide intuitive interpretations of SPFs, i.e., the change in crash frequency with one-unit increase in traffic volume or segment length. Note that, for ease of interpretation, AADT and segment length are scaled: the unit increment is 1,000 vehicles for traffic volume, and 1 mile for segment length. For standard SPFs with only one set of parameters (*a*, *b* and *c*), the average marginal effects for traffic volume and segment length were calculated by Equations (7) and (8):

$$MFX_{AADT} = \frac{1}{n}\sum_{i=1}^{n}(\mathrm{e}^{a+b\times\ln(AADT_i+1000)+c\times\ln(L_i)} - \mathrm{e}^{a+b\times\ln(AADT_i)+c\times\ln(L_i)}) \quad (7)$$

$$MFX_{Seg.\ Len.} = \frac{1}{n}\sum_{i=1}^{n}(\mathrm{e}^{a+b\times\ln(AADT_i)+c\times\ln(L_i+1)} - \mathrm{e}^{a+b\times\ln(AADT_i)+c\times\ln(L_i)}) \quad (8)$$

GWNBR generated SPFs, i.e., L-SPFs, included *n* sets of parameters $a(u_i, v_i)$, $b(u_i, v_i)$, and $c(u_i, v_i)$, where n is the number of segments investigated in this study. The marginal effects were computed for each segment:

$$MFX_{AADT,\ i} = \frac{1}{w_{ij}}\sum_{j=1}^{m}(w_{ij} \times (\mathrm{e}^{a_i+b_i\times\ln(AADT_{ij}+1000)+c_i\times\ln(L_{ij})} - \mathrm{e}^{a_i+b_i\times\ln(AADT_{ij})+c_i\times\ln(L_{ij})}) \quad (9)$$

$$MFX_{Seg.\ Len.,\ i} = \frac{1}{w_{ij}}\sum_{j=1}^{m}(w_{ij} \times (\mathrm{e}^{a_i+b_i\times\ln(AADT_{ij})+c_i\times\ln(L_{ij}+1)} - \mathrm{e}^{a_i+b_i\times\ln(AADT_{ij})+c_i\times\ln(L_{ij})}) \quad (10)$$

1.3.5. Model Comparison

To compare model performance of traditional NB model and GWNBR models, the log likelihood of models were calculated, as well as associated Pseudo-R^2 and AIC (Akaike Information Criterion). AIC has been often used for comparing geographically weighted models with associated traditional models (*28*). A smaller AIC estimate indicated a greater goodness-of-fit (*29*). Empirically, a three point decrease in the AIC magnitude implies a substantial improvement in model goodness-of-fit (*29*). The NB model's log likelihood at convergence can be obtained by calculating:

$$\ln L_{NB} = \sum_{i=1}^{n}(\ln(\Gamma(\frac{1}{\alpha} + y_i)) - \ln(\Gamma(y_i + 1)) - \ln(\Gamma\left(\frac{1}{\alpha}\right) + \frac{1}{\alpha}\ln\left(\frac{1}{1+\alpha\hat{y}_i}\right) + y_i \ln(1 - \frac{1}{1+\alpha\hat{y}_i})) \tag{11}$$

Where α is the NB over-dispersion parameter; y_i is the observed crash frequency on *i^th^* road segment; $\hat{y}_i$ is the expected crash frequency on *i^th^* road segment. The NB model's log likelihood at constant (or intercept-only model), $\ln L_{NB,\ intercept}$, can be obtained by replacing the $\hat{y}_i$ with the overall mean of observations $\hat{Y}$. For GWNBR, the log likelihood at convergence can be obtained by:

$$\ln L_{GWNBR} = \sum_{i=1}^{n}\sum_{j=1}^{m}(w_{ij} \times (\ln(\Gamma(\frac{1}{\alpha_i} + y_{ij})) - \ln(\Gamma(y_{ij} + 1)) - \ln(\Gamma\left(\frac{1}{\alpha_i}\right) + \frac{1}{\alpha_i}\ln(\frac{1}{1 + \alpha_i\hat{y}_{ij}})) \tag{12}$$

Where $w_{ij} = w_j(u_i, v_i)$; α_i is the NB over-dispersion parameter for local NB model at location (u_i, v_i); y_{ij} is the observed crash frequency for *j^th^* observation within the subgroup, which is centered at *i^th^* road segment in the overall sample, i.e. at location (u_i, v_i); $\hat{y}_{ij}$ is the expected crash frequency at location (u_i, v_i). For each subgroup, there are m observations, $m \leq n$. The $\ln L_{GWNBR,\ intercept}$, can be obtained in the same way based on Equation (12). The Pseudo-R^2, indicating explained deviance in percent, can be obtained by:

$$Pseudo\ R^2 = 1 - \frac{\ln L_{NB}}{L_{NB,\ intercept}} \quad \text{or} \quad 1 - \frac{L_{GWNBR}}{L_{GWNBR,\ intercept}} \tag{13}$$

For traditional NB model and GWNBR models respectively. Greater Pseudo-R^2 indicates a better goodness-of-fit. AIC (Akaike Information Criterion) can be obtained by:

$$AIC = -2\ln L_{NB} + 2k \text{ or } -2\ln L_{GWNBR} + 2k \quad (14)$$

Where *k* is the number of parameters in the model.

In addition, the non-stationary test was performed to compare the model estimates from NB and GWNBR models (*28*). The test is conducted to examine whether GWNBR estimates, $a(u_i, v_i)$, $b(u_i, v_i)$ and $c(u_i, v_i)$, vary significantly across the space. For this, the differences between the upper quartile and lower quartile of estimates from the GWNBR modeling are calculated, along with the resulting significance of estimated parameters:

$$Delta = \beta^{upper} - \beta^{lower} \begin{cases} > 1.96\,(S.E.) \ \textit{and} \ \text{Max.}\,|z| > 1.96, & \text{Pass the Non} - \text{stationarity Test} \\ \text{otherwise,} & \text{Fail the Non} - \text{stationarity Test} \end{cases} \quad (15)$$

Where $S.E.$ is the standard error of the estimates in traditional NB model, and $|z_i|$ is the significance z-value of the GWNBR estimates at location(u_i, v_i). If $Delta$ was more than $1.96\ (S.E.)$ and $\text{Max.}\,|z|$ is greater than 1.96, then the non-stationarity test is passed, indicating that the GWNBR estimates, $a(u_i, v_i)$, $b(u_i, v_i)$ and $c(u_i, v_i)$, vary substantially in space (*28*); otherwise, GWNBR estimates are close to traditional or global NB estimates (indicating that L-SPFs do not provide additional information than traditional SPFs).

1.4. MODELING & MAPPING RESULTS

Table 2 presents the modeling results of the traditional NB model (for standard SPF) and GWNBR model (for L-SPFs). The NB model estimated one set of parameters *a*, *b* and *c* for the whole study area, and these parameters were assumed to be stationary within the study area. The GWNBR model resulted in an individual set of parameters for each segment, and the parameters were allowed to vary substantially across the study area.

Both the NB and GWNBR models demonstrate a reasonable goodness-of-fit, as indicated by the significantly increased log-likelihood from intercept-only models as well as the sizable Pseudo R^2 (percent of explained deviations). The GWNBR models were found to have a better goodness-of-fit, in terms of the smaller AIC value (reduction >3) and greater Pseudo R^2 (*28*) (*29*). Through the non-stationarity tests, all three parameters have substantial spatial variation. All the test-statistics imply that GWNBR models better explain the associations between traffic volume, segment length and crash frequency than NB model does, and the SPFs localized in space better reflect the local attributes of crash outcomes and they are significantly different from the standard SPF.

Table 2 Estimation Results of Negative Binomial and Geographically Weighted Negative Binomial Regressions

Parameter			Intercept	Ln (AADT)	Ln (Seg. Length)	Alpha
	NB (Standard SPF)	Estimate	-12.694	1.331	0.795	0.499
		Std. Err.	0.331	0.030	0.024	0.384

		z-value	-38.302	44.362	33.703	-
		p-value	0.000	0.000	0.000	0.000
	GWNBR (Localized SPFs)	Mean Est.	-13.022	1.350	0.785	0.438
		Minimum Est.	-15.975	0.781	0.624	0.228
		Maximum Est.	-7.032	1.627	0.900	0.714
		Lower Est.	-15.587	0.924	0.690	0.266
		Upper Est.	-8.495	1.591	0.874	0.637
		Delta	7.092	0.667	0.184	-
		Min. |z|	7.077	7.904	8.311	-
		Max. |z|	18.986	21.556	18.580	-
	Non-Stationarity Test		YES	YES	YES	-
Summary Statistics					Standard SPF	Localized
		Number of Observations			2116	2116
		Log-Likelihood at Intercept-only Model			-5362.583	-5362.583
		Log-Likelihood at Regression Model			-3797.928	-3716.559
		Pseudo R^2			0.292	0.307
		AIC			7603.856	7441.118
Marginal Effects			AADT (per 1,000 veh.)		Segment Length (per 1 mile)	
	Standard SPF	MFX	0.142		4.111	
		Std. Dev.	0.164		6.272	
	Localized SPF	Mean MFX	0.141		4.216	
		Minimum MFX	0.107		1.409	
		Maximum MFX	0.216		13.216	
		Lower MFX	0.114		1.661	
		Upper MFX	0.155		5.203	

Notes: *Delta* = $Upper\ Est. - Lower\ Est.$; YES = *Delta* > 1.96 *Std. Err.* and the non-stationarity test was passed.

The parameter estimates of NB model and GWNBR models are in general reasonable and consistent with a study that developed negative binomial SPFs for Virginia's freeway segments (*2*). Both traffic volume and segment length are positively correlated with crash frequency. The mean marginal effects (Table 2) of parameters in standard SPFs show that an increase of 1,000 vehicles is associated with a 0.14 increase in crash frequency. Whereas, a one-mile increase in segment length is associated with an increase of 4.11 in crash frequency. Note that parameter *c* for segment length was estimated to be less than 1, indicating that the crash frequency is not proportionally related to segment length (doubling the segment length does not mean the crash frequency will also be doubled). Being less than 1 implies that a longer homogenous road segment is associated with a smaller number of crashes per unit segment length. In other words, if the conditions of road segment change less frequently along a way, less crashes per unit length may be expected.

The GWNBR-estimated parameters have ranges that include the estimates in the NB model. The NB model estimates are close to the mean estimates in GWNBR models. Noticeably, all estimates in GWNBR models are statistically significant ($|z| > 1.96$). The estimates of parameter *a* range from -15.587 to -7.032, parameter *b* has a range from 0.781 to 1.627, and parameter *c* ranges from 0.624 to 0.9. Figure 2 presents the varying parameters along freeways in Virginia. The three maps in Figure 2 together provide L-SPFs for the entire Commonwealth of Virginia. The Localized parameters can vary across segments. These estimates are relatively stationary in certain regions (but not constrained within any specific jurisdictions). For example, the parameter *a* for the intercept has a relatively stable estimate around -8.3 in the southwest of Virginia, the estimates are greater -7 in the mid-west of Virginia (Roanoke, Lexington and Staunton), and east of Virginia (Richmond - Norfolk) was found to have relatively stable estimates lower than -15.

The L-SPFs from GWNBR models show that the parameter for AADT has the greatest estimate in east of Virginia (Norfolk), around 1.60, while the Mid-west of Virginia has the smallest estimate around 0.80. It implies that the association of AADT with crashes on freeway segments is stronger in Eastern Virginia than Western Virginia. This may be attributed to various factors that differ in these two regions, such as the different driving cultures because there is a natural barrier (Blue Ridge Mountains) between these two regions. The parameters for segment length were found to have greater values in both north and south parts of Virginia, with a magnitude around 0.90. In the capital region of Richmond, these parameters are around 0.65, which are smaller than in other regions, indicating that the length of homogenous freeway segment has a smaller association with crashes.

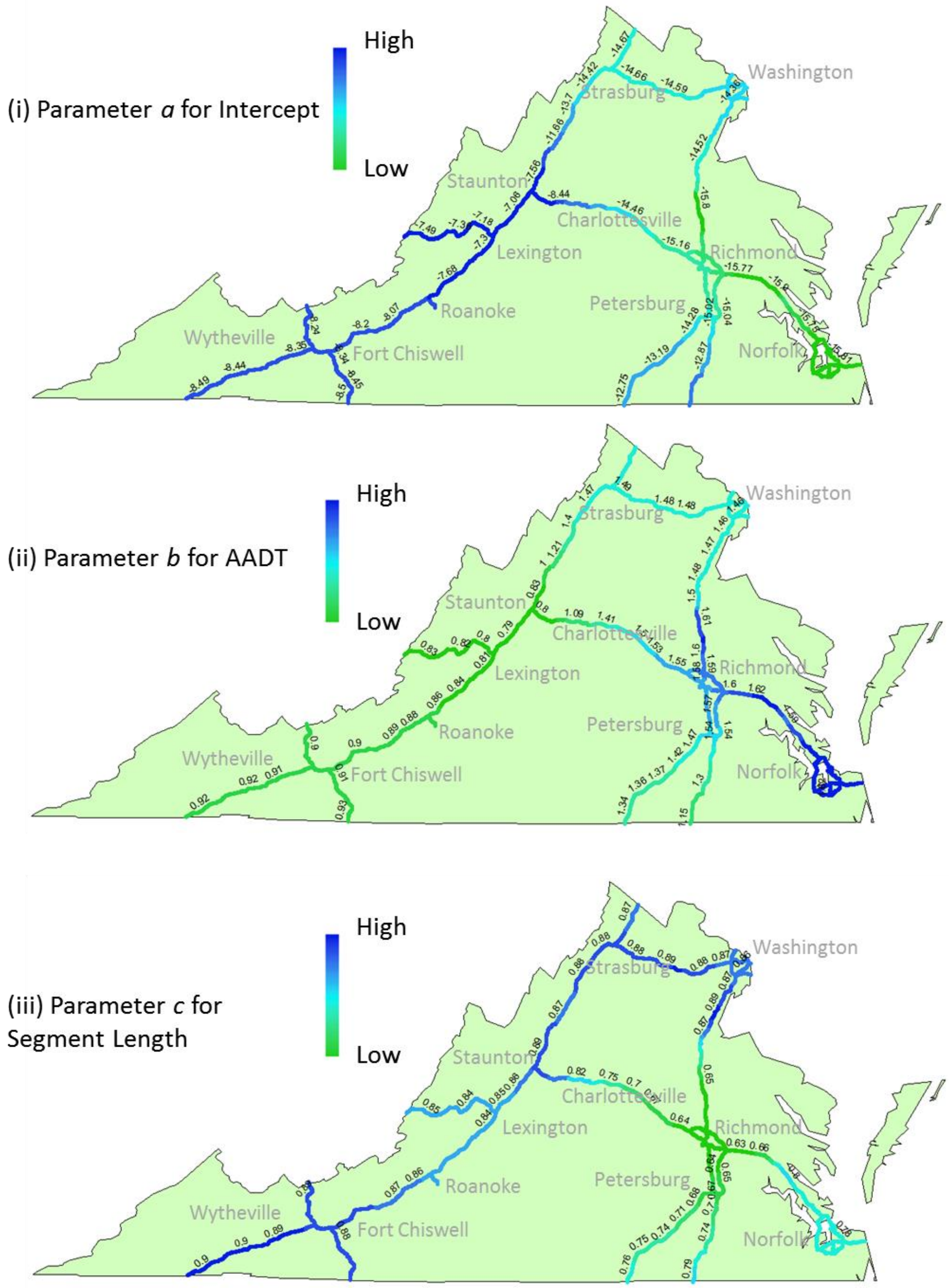

Figure 2 Visualization of parameter estimates obtained from GWNBR model

1.5. LIMITATIONS

Currently, there is no detailed methodology (to the best of our knowledge) for predicting crash frequencies at freeways that is officially included in HSM. This precludes the comparison of GWNBRs developed in this study with HSM SPFs as there is no HSM SPF for freeways. However, the key point is that SPFs can be easily localized using the proposed spatial methodology. Certainly spatial regressions cannot be used for crash frequency predictions at other locations, e.g., we cannot transfer the model estimated in Virginia to Tennessee or North Carolina (unless we find that there is no significant spatial variation). This said, each state will have to apply the proposed framework for their entire network for a specific roadway type; but this is doable, given the computational power of the tools available. Furthermore, this study has focused on spatial/geographic stability within a state, but did not address temporal stability of SPFs. Future research may examine spatial stability across states (see Farid et. al (*19*). Overall, GWNBR models are highly localized and they are not generalizable to other states, i.e., if the GENBR methodology is adopted by a state, then they will have to develop their own localized SPFs.

1.6. CONCLUSIONS & IMPLICATIONS

The current study has focused on an important methodological concern related to geographic stability assumed in the global or traditional negative binomial models. To localize SPFs in HSM, this study successfully demonstrates the application of localized models known as Geographically Weighted Negative Binomial Regressions. Typically, stability of the associations between crash frequencies and various factors is assumed. Given that crash data are typically location-referenced, it is possible to use geo-referencing along with new spatial analysis techniques to capture and quantify spatial stability. By using 2013 geo-referenced freeway crash data from Virginia, this study estimated highly localized SPFs using GWNBR. L-SPFs better reflect the local conditions of roadways than jurisdiction-specific SPFs, as their estimations consider the spatial heterogeneity among roadway segments. L-SPFs are estimated in continuous space at the roadway segment level and they are not constrained by jurisdiction boundaries. A unique aspect of the study is that we can potentially take advantage of large-scale geo-referenced accident data available for most states and apply new spatial analysis techniques as well as computational power to provide "customized" information about expected crashes and their relationships with exposure variables.

Collectively, GWNBR models revealed that the parameters of freeway segment SPFs vary substantially across the Commonwealth of Virginia, indicating the low transferability of statewide SPF, which highlights the importance of developing L-SPFs. In general, the varying parameters of L-SPFs estimated by GWNBR models have ranges that include the stationary parameters in a traditional or global SPF estimated from a NB model. A standard statewide SPF estimated in the study is $N_{SPF} = \mathrm{e}^{12.694+1.331\times\ln(AADT)+0.795\times\ln(L)}$, while the three parameters in L-SPFs have fairly wide ranges (-15.975, -7.032), (0.781, 1.627), and (0.624, 0.9), respectively. While the parameters in L-SPFs can vary substantially across one segment to another segment, the GWNBR models visualized as maps revealed that estimates are relatively stationary in certain regions of Virginia (these are organized spatially and not within any specific jurisdictions). Thus there are some regions where associations between crash outcomes and contributing factors are relatively stationary. To summarize, the key findings include:

- Parameter *a* for the intercept. Southwest of Virginia has a relatively stable estimate around -8.3. In the Mid-west region of Virginia (Roanoke, Lexington and Staunton), the estimates are around -7. And in the Southeastern Virginia (Richmond - Norfolk) the estimates are lower, around -15.
- Parameter *b* for AADT. Southeastern Virginia (Hampton Roads area) has a larger estimate around 1.60, while the Mid-west Virginia region estimates are much smaller than other regions, at around 0.80. This result implies that the association of AADT with crash frequency on freeway segments is higher in Southeastern Virginia than Southwestern Virginia, which may be related to the different local conditions in these two regions separated by a natural barrier (Blue Ridge Mountains).

- Parameter *c* for segment length. Both north and south parts of Virginia have larger estimates, around 0.90, than other regions. Richmond area have the smallest estimate at around 0.65. This implies that the length of homogenous freeway segment has a stronger association with crash frequency in this region.

The results obtained from this study have important safety implications. First, GWNBR provides mechanisms for making base crash frequency predictions that are customized to specific locations. A key advantage of L-SPFs is that more realistic estimates of crashes can be obtained that can better identify hazardous sites, and more appropriate countermeasures can be developed. Second, given that base crash frequencies can be predicted, the proposed spatial framework has the potential to change the current HSM state-of-the-practice, i.e. calibration of global or traditional Negative Binomial models used in HSM may not be needed. Compared to HSM which uses both Empirical Bayes (history of crashes and global SPFs for a site), the proposed methodology can potentially replace the global SPFs in HSM with a local SPF. Third, the practical aspects are the potential changes in the practices of HSM. Although the proposed methodology is complex compared with current practices, it systematically accounts for spatial variations and given this methodology, practitioners can have better tools to undertake more informed decisions. The software application is under development, in order to provide tools to various transportation agencies who are interested in developing their own L-SPFs with various specifications (i.e., different contributing factors) using their region's crash data.

Finally, the long-term value of this study comes from the idea of spatial heterogeneity, i.e. estimating local versus global models that can be applied for prediction of crashes. Although we found evidence of significant spatial heterogeneity within Virginia, at least for freeways, this does not mean that other states will have similar results. However, the proposed methodology can be applied in other states to obtain more customized and localized estimates of crash frequencies. Within this premise, large databases such as HSIS can be used to explore the spatially varying relationships for different states in the future. Though this study strictly followed the SPF development procedures in HSM by analyzing only the associations between traffic volume, segment length, and crash frequency (based on the NB framework), the method proposed in this study can be easily expanded to explore more complex spatial patterns of associations between crash frequency and other contributing factors such as shoulder, lane width, and other geometric characteristics, and for different roadway types.

2. EXPLORING NON-LINEAR DEPENDENCIES IN CORRELATES OF ROADWAY CRASHES[2]

ABSTRACT – For practical considerations and in many cases the difficulty to collect detailed crash-related data, Annual Average Daily Traffic (AADT) and segment length are often used as the main correlates for predicting crash frequencies on segments. Typically, crash frequencies are assumed to linearly depend on traffic exposure related factors which may not realistically represent the underlying complexity embedded in crash data generated by physical and social elements of transportation systems. Thus, the objective of the current study is to investigate and quantify nonlinear dependencies of crash frequency on traffic exposure related factors. Using crash data collected on rural two-lane two-way roads in Tennessee, total crashes and total injury crashes were modeled using Negative Binomial Generalized Additive Models (NBGAMs) that are well-suited for conceptualizing non-linear relationships. In addition, including too few explanatory factors (such as AADT and segment length only) in crash frequency modeling may lead to omitted variable bias, and in such cases the nonlinearity may be an outgrowth of missing information on important variables. To address this issue, additional data on important correlates are collected and incorporated in NBGAM framework. The modeling results show that the relationship between crash frequencies (total crashes and total injury crashes) and AADT is clearly non-linear. Importantly, the non-linear dependency of crash frequencies on segment length is more complex than its dependence on AADT. The goodness of fit measures indicates the promising potential of NBGAMs in approximating non-linear dependencies of crash frequencies on associated factors. Important practical implications of results are presented with respect to rural two-lane two-way road safety.

Note: The full-length paper is available from authors.

[2] Abstract is based on Khattak A., B. Wali, X. Li, Exploring Non-Linear Dependencies in Correlates of Roadway Crashes. To be submitted to a transportation conference and a safety journal for publication review.

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